The LV-Category (and LV-module) for $SU(n,1)$, $n \geq 1$

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Research group (UNSW)

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Overview

we do everything for $SU(n,1)$

1. Computing the LV-module and hom pairing
   - Recap: Hecke algebra and LV-module
   - Computing the LV-module
   - The hom pairing

2. LV category diagrammatics
   - Recap: Soergel bimodule diagrammatics
   - LV category examples
   - Light leaves & independence

1. Computing the LV-module & hom pairing

<u>Recap</u>

- Coxeter System (W,S)

$$W = \langle s_1, \dots, s_n \mid (s_i s_j)^{m_{s_i s_j}} = 1 \rangle, \quad S = \{s_1, \dots, s_n\}$$

$\rightsquigarrow$ Hecke algebra $\mathcal{H} = \langle \delta_{s_1}, \dots, \delta_{s_n} \mid \delta_{s_i}^2 = (v^{-1} - v)\delta_{s_i} - 1, \ \underbrace{\delta_{s_i}\delta_{s_j}\cdots}_{m_{s_i s_j}} = \underbrace{\delta_{s_j}\delta_{s_i}\cdots}_{m_{s_i s_j}} \rangle$

"standard basis" on $\mathbb{Z}[v^{\pm 1}]$

Kazhdan-Lusztig basis $\{b_{s_i}\}_i$ over $\mathbb{Z}[v^{\pm 1}]$

- self dual in $\mathbb{Z}$-linear involution: $\bar{v} = v^{-1}$, $\bar{\delta}_w = \delta_{w^{-1}}^{-1}$
- upper triangular in standard basis w/ entries $\in v\mathbb{Z}[v]$ & 1's on diagonal

1. Computing the LV-module & hom pairing

Recap

"real Lie group"

- For the LV-module: take real form $(G_\mathbb{C}, \sigma)$
  1. fix Borel $B \subset G_\mathbb{C}$ ⇝ weyl group (W,S) (a Coxeter group)
  2. pick $K \subset G_\mathbb{C}^\theta$ w/ finite index

     ⇝ $K \curvearrowright G/B$ action on flag variety ; $W_K \leq W$

     ⇝ $\mathcal{D} = \{(\mathcal{O},\mathcal{L}) : \mathcal{O} = K\text{-orbit},\ \mathcal{L} = K\text{-equivariant local system on } \mathcal{O}\}$

     standard basis

     ⇝ LV module

     $M = \langle \mathcal{D} \rangle \curvearrowleft \mathcal{H}(W,S)$

     └ geometric & nasty

$$\begin{tikzcd} & G_\mathbb{C} \arrow[dl, "\theta"'] \arrow[dr, "\sigma"] & \\ K \subset G_\mathbb{C}^\theta & & G_\mathbb{R} \end{tikzcd}$$

holo. invol.

antiholo. invol.

- Lusztig & Vogan: There is a "KL"-basis $\{\underline{\delta} : \delta \in \mathcal{D}\}$
  - self dual in the involution D

    ↑ def: unique D respecting $\mathcal{H}$ action and similar degree bound property as KL-basis
  - upper triangular in standard basis w/ entries $\in v\mathbb{Z}[v]$ & 1's on diagonal

We call this the KLV-basis.

1. Computing the LV-module & hom pairing

For $SU(n,1)$, $S = \{s_1, s_2, \dots, s_n\}$

$W = S_{n+1}$ symmetric group

$W_K = \langle s_1, \dots, s_{n-1} \rangle \leq W$

$\cong S_n$

<u>**Q**</u> How can we know what the LV module looks like?

(without much geometry)

1. Computing the LV-module & hom pairing

$\boxed{SU(n,1)}$ $S=\{s_1,\dots,s_n\}$, $W=S_{n+1}$, $W_K=\langle s_1,\dots,s_{n-1}\rangle\subset W$

Q. How can we know what the LV module looks like?

① use `atlas` ("Alas of lie groups and Representations") software

⇝ KLV-basis via

```
print_KL_basis(blocks(G)[i])
```

(need to renormalise by conjugating with diagonal matrix $(v^{\ell(s)})_{s\in\mathcal{D}}$)

⇝ W-graph via

```
print_KGB(G)
print_W_graph(block of G)
```

1. Computing the LV-module & hom pairing

$\boxed{SU(n,1)}$ $S=\{s_1,\dots,s_n\}$, $W=S_{n+1}$, $W_K=\langle s_1,\dots,s_{n-1}\rangle\subset W$

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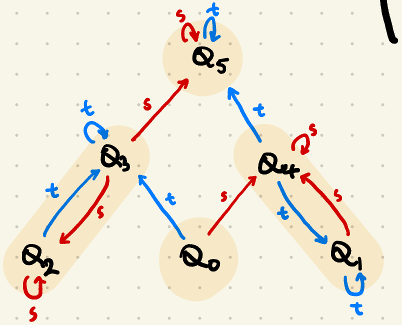
print_W_graph(block of G)
```

eg. SU(2,1)

$$\begin{pmatrix}1&&&&&\\&1&&&&\\&&1&&&\\&&&v^{-1}&&\\&&&&v^{-1}&\\&&&&&v^{-2}\end{pmatrix}\left(\begin{array}{cccccc}Q_0&Q_1&Q_2&Q_3&Q_4&Q_5\\1&0&0&1&1&1\\&1&0&0&1&1\\&&1&1&0&1\\&&&1&0&1\\&&&&1&1\\&&&&&1\end{array}\right)\begin{pmatrix}1&&&&&\\&1&&&&\\&&1&&&\\&&&v&&\\&&&&v&\\&&&&&v^2\end{pmatrix}$$

$$=\begin{pmatrix}1&0&0&v&v&v^2\\&1&0&0&v&v^2\\&&1&v&0&v^2\\&&&1&0&v\\&&&&1&v\\&&&&&1\end{pmatrix}$$



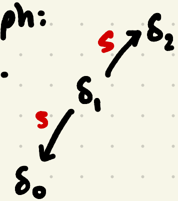
1. Computing the LV-module & hom pairing

$\boxed{SU(n,1)}$ $S=\{s_1,\dots,s_n\}$, $W=S_{n+1}$, $W_K=\langle s_1,\dots,s_{n-1}\rangle\subset W$

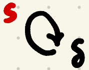
Q. How can we know what the LV module looks like?

② FACT for SU(n,1) we can read action of KL-basis in $\mathcal{H}$ on KLV-basis in M from W-graph:

eg.



$\Rightarrow$ $\underline{\delta_1}\cdot b_s=\underline{\delta_0}+\underline{\delta_2}$



$\Rightarrow$ $\underline{\delta}\cdot b_s=(v+v^{-1})\underline{\delta}$

We know $b_s=\delta_s+v$ in $\mathcal{H}$ and KLV-basis $\{\underline{\delta}\}_{\delta\in\mathcal{D}}$ (in terms of standard basis)

lin. alg. $\Rightarrow$ eg. Sage-math we know $\gamma\cdot\delta_s$, for $\gamma\in\mathcal{D}$, $\delta_s\in\mathcal{H}$, $s\in S$.

↑ all the info of the module

1. Computing the LV-module & hom pairing

"Hom" pairing in M

Want: $(\cdot,\cdot) : M \times M \to \mathbb{Z}[v^{\pm 1}]$ s.t.

$$(X \cdot b_s , Y) = (X, Y \cdot b_s) \text{ for all } s \in S$$

"b_s self-dual"

Answer:

- have a "trace" $\langle \cdot, \cdot \rangle : M \times M \to \mathbb{Z}[v^{\pm 1}]$
- the "standard" pairing in

$$(\cdot,\cdot) : M \times M \to \mathbb{Z}[v^{\pm 1}]$$
$$(X, Y) \mapsto \langle D(X), Y \rangle$$

Compare w/ $\mathcal{H}$

- Trace: $\mathbb{Z}[v^{\pm 1}]$-linear map

$$\varepsilon : \mathcal{H} \to \mathbb{Z}[v^{\pm 1}]$$
$$\delta_{id} \mapsto 1$$
$$\delta_x \mapsto 0 \quad \text{for } x \neq id$$

- Standard pairing: sesquilinear form

$$(\cdot,\cdot) : \mathcal{H} \times \mathcal{H} \to \mathbb{Z}[v^{\pm 1}]$$
$$(a,b) \mapsto \varepsilon(\underset{\uparrow}{w}(a)\, b)$$

anti-involution
$w(\delta_x) = \delta_x^{-1}$

1. Computing the LV-module & hom pairing

"Hom" pairing in M

DEF ("Trace")

Symmetric bilinear pairing $\langle \cdot,\cdot \rangle : M \times M \to \mathbb{Z}[v^{\pm 1}]$ defined inductively

- If $X \neq Y$ then $\langle X, Y \rangle = 0$
- otherwise follow an increasing path in W-graph

$$w_0 \xrightarrow{s_1} w_1 \to \cdots \xrightarrow{s_n} w_n = X$$

where $\dim(w_0) = 0$. Then

$$\langle X, X \rangle := \prod_{i=0}^{n-1} \alpha^{s_i}_{w_{i+1}, w_i}$$

$\alpha^{s}_{X,Y}$ = coeff of Y in $X \cdot \delta_s$

This construction came from assuming $\langle X, Y \cdot b_s \rangle = \langle X \cdot b_s, Y \rangle$ (see next example)

DEF

Sesquilinear pairing $(\cdot,\cdot) : M \times M \to \mathbb{Z}[v^{\pm 1}]$

$$(X, Y) \mapsto \langle D(X), Y \rangle.$$

CONJECTURE In $\mathcal{N}$, write $[B] = \sum_{\delta \in \mathcal{D}} c_\delta \delta$, $[B'] = \sum_{\delta \in \mathcal{D}} c'_\delta \delta$. Then

$$\mathrm{rk}' \operatorname{Hom}^\bullet(B, B') = ([B], [B]') = \sum_{\delta \in \mathcal{D}} c_\delta c'_\delta \langle \delta, \delta \rangle$$

Compare w/ $\mathcal{H}$

- Trace: $\mathbb{Z}[v^{\pm 1}]$-linear map

$$\varepsilon : \mathcal{H} \to \mathbb{Z}[v^{\pm 1}]$$
$$\delta_{id} \mapsto 1$$
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- Standard pairing: sesquilinear form

$$(\cdot,\cdot) : \mathcal{H} \times \mathcal{H} \to \mathbb{Z}[v^{\pm 1}]$$
$$(a, b) \mapsto \varepsilon(\omega(a)\, b)$$

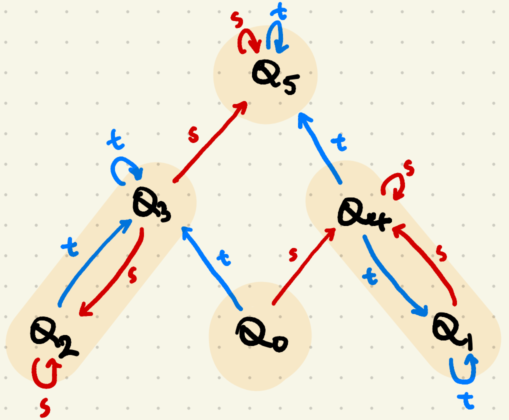
anti-involution $\omega(\delta_x) = \delta_x^{-1}$

1. Computing the LV-module & hom pairing

"Hom" pairing in M

eg. $SU(2,1)$

$$
\begin{aligned}
\langle Q_5, Q_5\rangle &= \langle Q_4 \cdot b_t , Q_5 \rangle \\
&= \langle Q_4, Q_5 \cdot b_t \rangle \\
&= \langle Q_4, Q_5 \cdot \delta_t \rangle \\
&= \langle Q_4, Q_4 \rangle \\
&= \langle Q_0 \cdot b_s , Q_4 \rangle \\
&= \langle Q_0 , Q_4 \cdot b_s \rangle \\
&= \langle Q_0, Q_4 \cdot \delta_s \rangle \\
&= \langle Q_0, Q_0 \rangle \, (v^{-1}-v)v \\
&= 1-v^2
\end{aligned}
$$



Just enough facts

$$
\begin{cases}
Q_5 \cdot \delta_t = (v-v^{-1})\, Q_5 + Q_4 \\
Q_4 \cdot \delta_s = (v^{-1}-2v)\, Q_4 + (v^{-1}-v)v\,(Q_2+Q_0)
\end{cases}
$$

- $\underline{\delta} \in \delta + \sum_{\gamma<\delta} v\mathbb{Z}[v]\,\gamma$
- $b_s = \delta_s + v \quad , \ \forall s \in S$

1. Computing the LV-module & hom pairing

"Hom" pairing in M

eg. $SU(2,1)$

Trace $\langle Q_0, Q_0\rangle = 1$

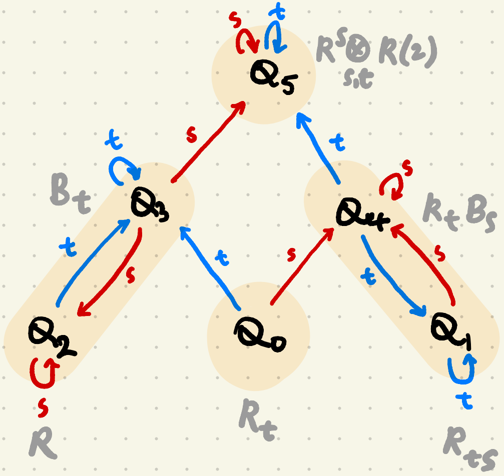
$\langle Q_1, Q_1\rangle = 1$

$\langle Q_2, Q_2\rangle = 1$

$\langle Q_3, Q_3\rangle = 1-v^2$

$\langle Q_4, Q_4\rangle = 1-v^2$

$\langle Q_5, Q_5\rangle = 1-v^2$



Hom pairing on indecomposables

	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5
Q_0	1	0	0	v	0	v^2
Q_1	0	1	0	v	v	v^2
Q_2	0	0	1	0	v	v^2
Q_3	v	v	0	$1+v^2$	v^2	$v+v^3$
Q_4	0	v	v	v^2	$1+v^2$	$v+v^3$
Q_5	v^2	v^2	v^2	$v+v^3$	$v+v^3$	$1+v^2+v^4$

1. Computing the LV-module & hom pairing

Computing the "Hom" pairing on KLV-basis

- already know $\delta \cdot b_s$ for $\delta \in \mathcal{D}$, $s \in S$ & W-graph

  ⇝ know $\langle \cdot, \cdot \rangle$

- already know KLV basis ⟷ std. basis
- since $D(\underline{\delta}) = \underline{\delta}$ $\forall \delta \in \mathcal{D}$, we also know

$$
\begin{aligned}
(\underline{\delta}, \underline{\delta}') = \langle D(\underline{\delta}), \underline{\delta}' \rangle &= \langle \underline{\delta}, \underline{\delta}' \rangle \\
&= \Big\langle \sum_{\gamma \in \mathcal{D}} c_\gamma \gamma , \sum_{\gamma \in \mathcal{D}} c'_\gamma \gamma \Big\rangle \\
&= \sum_{\gamma \in \mathcal{D}} c_\gamma c'_\gamma \langle \gamma, \gamma \rangle
\end{aligned}
$$

2. LV-category diagrammatics

__Recap__ For each M, $\swarrow (W,S),\ W_K \leq W$ we can categorify the trivial block of M to get

$$\mathcal{N} := \mathcal{N}^0 \curvearrowleft \text{SBim} \quad \xrightarrow{K_0} \quad M^0 \curvearrowleft \mathcal{H}$$

Concretely define

$$\tilde{\mathcal{N}} = \langle R_w \ : w \in W_K \backslash W \rangle \curvearrowleft \text{BSBim}$$

as a full subcategory of (R^{W_K}, R)-bimodules
then

$$\mathcal{N} \curvearrowleft \text{SBim} = \text{Kar}(\tilde{\mathcal{N}} \curvearrowleft \text{BSBim}).$$

__DEF__ (Standard modules)

$R_w = R$ as left R-mod
and for $f \in R_w$, $r \in R$,
$f \cdot r = f w(r)$

- Restricting to R^{W_K},
$R_w = R_{w'} \iff W_K w = W_K w'$

2. LV-category diagrammatics

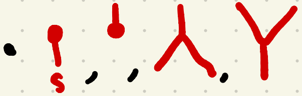
Recap Diagrammatic BSBim

Fix coxeter system (W,S). Define $\mathcal{D}$ to be the R-linear monoidal cat w/

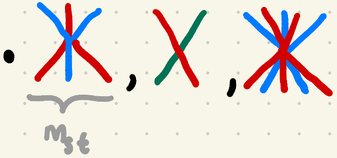
ob: finite tensors of generating symbols $s \in S$ ← think of as colours

eg. $\mathbb{1}$, s, ss, sss, sst, $ststsu$, etc.

mor: finite ← incl. identity morphisms composition & tensors of

- 

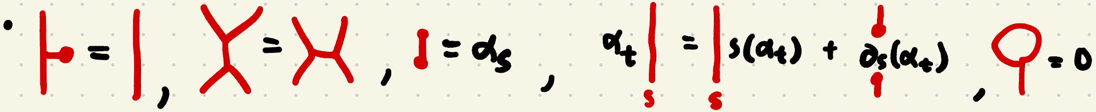
for each $s \in S$

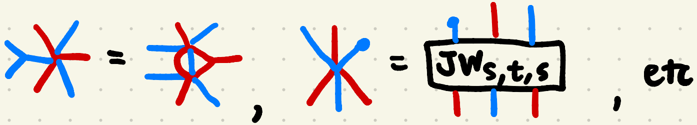
- 

for each $s,t \in S$ $s \neq t$

FACT (Elias-Williamson)
$\mathcal{D} \simeq$ BSBim
as monoidal cats

up to planar isotopy & local relations

- 

- 2 col relations eg. 

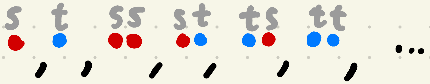
- 3 col relations

2. LV-category diagrammatics

Recap Diagrammatic BSBim

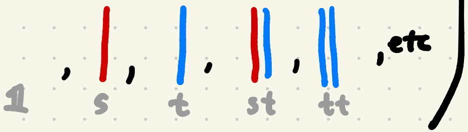
example: $W = S_3 = \langle s,t : s^2=t^2=1,\ sts=tst \rangle$, $S=\{s,t\}$

$\mathcal{D}$: ob $\mathbb{1}$,



mor composition & tensor generated by

- all identity morphisms



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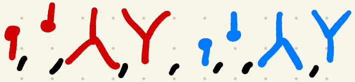

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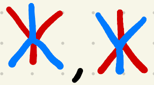

up to planar isotopy & relations

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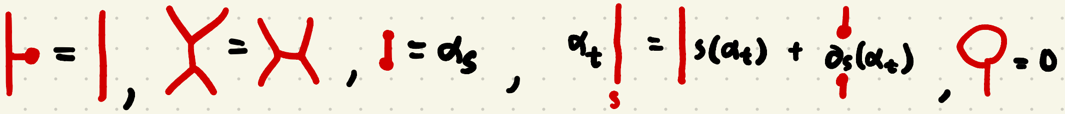

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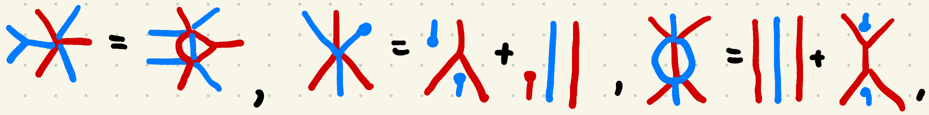


etc.

2. LV-category diagrammatics

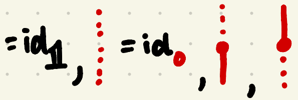
Diagrammatic $\tilde{\mathcal{N}}$ for $SU(1,1) \simeq SL(2,\mathbb{R})$ (revision)

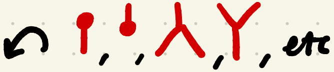
- start w/ $W = S_2$, $S = \{s\}$, $W_K = 1 \leq W$, $W_K \backslash W = W$
- Define $\mathcal{D}\tilde{\mathcal{N}}$ as right module cat over $\mathcal{D}$

ob gen by 1, ○ ↶ 1, ●, ●●, ...

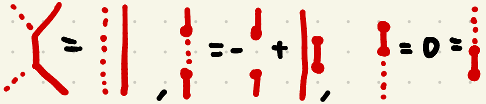
eg. 1, ●, ●●, ●●●, ..., ○, ○●, ○●●, ...

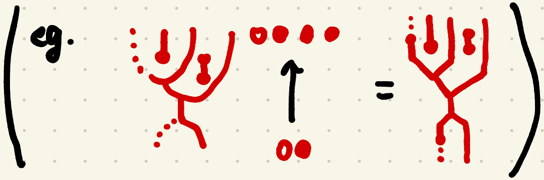
mor gen by





under limited isotopy & local relations





Warning! We have limited isotopy:
- solid strand never left of dotted
- o/w: usual isotopy of solid strand inherited from $\mathcal{D}$

2. LV-category diagrammatics

Diagrammatic $\tilde{N}$ for SU(2,1)

parabolic $S_K = \{s\}$

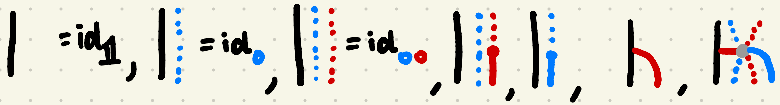
- Start w/ $W = S_3 = \langle s,t \mid s^2 = t^2 = 1,\ sts = tst \rangle$, $S = \{s,t\}$, $W_K = \langle s \rangle \leq W$, $W_K \backslash W = \{\emptyset, t, ts\}$
- Define $\mathcal{D}\tilde{N}$ as right module cat over $\mathcal{D}$

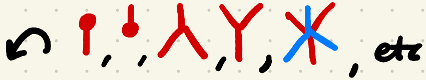
ob gen by $\mathbb{1}$, ○, ○○ $\quad\curvearrowleft\quad \mathbb{1}$, ●, ●, ●●, ●●

eg. $\mathbb{1}$, ●, ●●, ..., ○, ○●, ○●●, ...

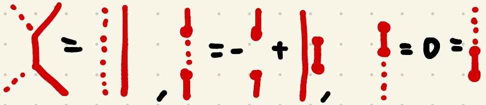
we keep track of the colours by $(\underline{x} \mid \underline{y})$: eg ○○●●● =: (ts|stt)

mor gen by



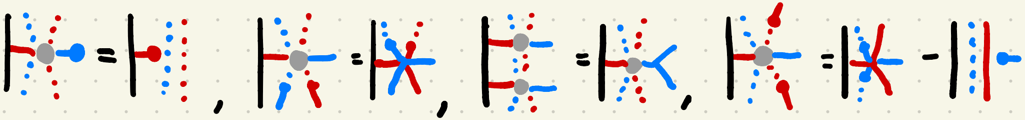


under limited isotopy & local relations



for each colour in S

for each colour in S_K



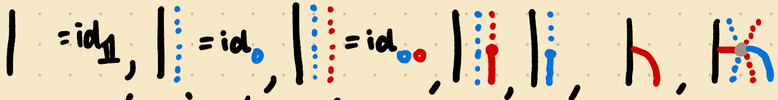
2. LV-category diagrammatics

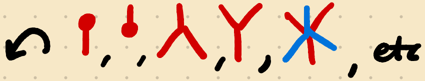
Diagrammatic $\tilde{N}$ for $SU(2,1)$

($W = S_3 = \langle s,t \mid s^2 = t^2 = 1,\ sts = tst\rangle$, $S = \{s,t\}$,
$W_K = \langle s\rangle \leq W$, $S_K = \{s\}$, $W_K\backslash W = \{\emptyset, t, ts\}$)

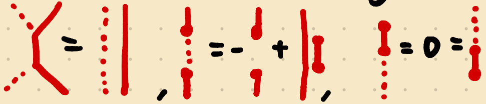
- $D\tilde{N} \curvearrowleft D$:

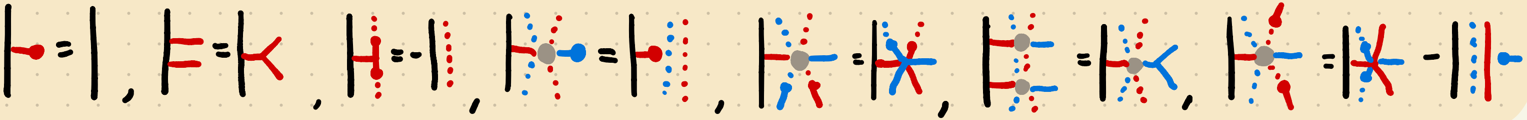
<u>Mor</u> gen by



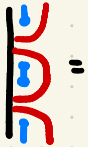


under limited isotopy & local relations





eg.  is an idempotent



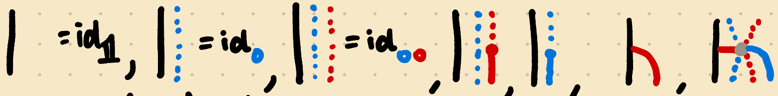
2. LV-category diagrammatics

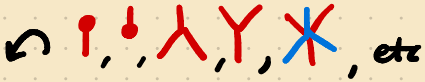
Diagrammatic $\tilde{\mathcal{N}}$ for SU(2,1)

$\big(W = S_3 = \langle s,t \mid s^2 = t^2 = 1,\ sts = tst \rangle,\ S = \{s,t\},$
$W_K = \langle s \rangle \leq W,\ S_K = \{s\},\ W_K\backslash W = \{\emptyset, t, ts\} \big)$

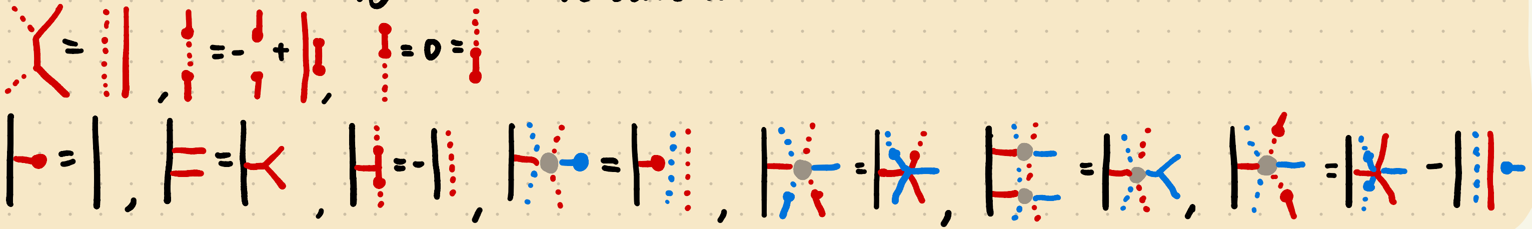
- $D\tilde{\mathcal{N}} \curvearrowleft \mathcal{D}$:

Mor gen by

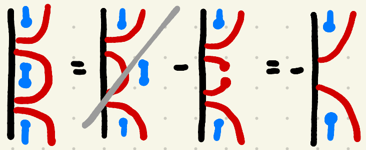




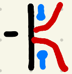
under limited isotopy & local relations

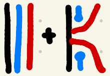


eg.  is an idempotent



$$\begin{tikzcd} 0 \arrow[r] & R^s \otimes_{t,s} R(2) \arrow[r, "1\otimes 1 \mapsto 1 \otimes_t 1 \otimes_s 1"] & B_t B_s \arrow[r] & R \arrow[r] \arrow[l, bend left] & 0 \end{tikzcd}$$

 : $B_t B_s \twoheadrightarrow R \hookrightarrow B_t B_s$

 : $B_t B_s \twoheadrightarrow R^s \otimes_{t,s} R(2) \hookrightarrow B_t B_s$

2. LV-category diagrammatics

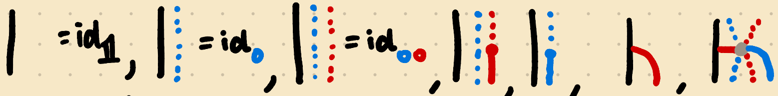
Diagrammatic $\tilde{\mathcal{N}}$ for $SU(2,1)$

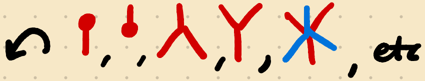
$\left(W = S_3 = \langle s,t \mid s^2 = t^2 = 1, sts = tst \rangle, \ S = \{s,t\}, \right.$

$\left. W_K = \langle s \rangle \leq W, \ S_K = \{s\}, \ {}^{W_K \backslash}W = \{\emptyset, t, ts\} \right)$

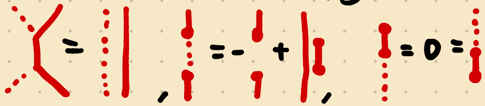
- $\mathcal{D}\tilde{\mathcal{N}} \curvearrowleft \mathcal{D}$:

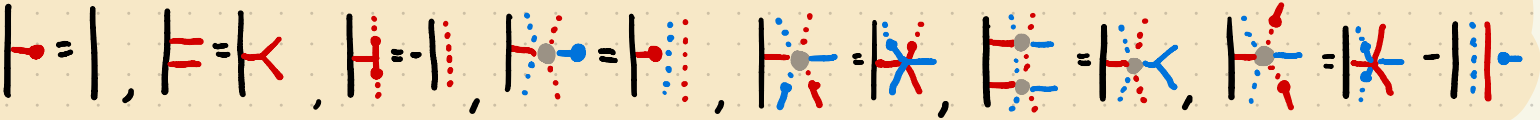
Mor gen by

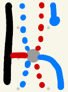


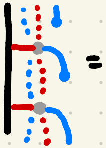


under limited isotopy & local relations





eg.  is an idempotent



2. LV-category diagrammatics

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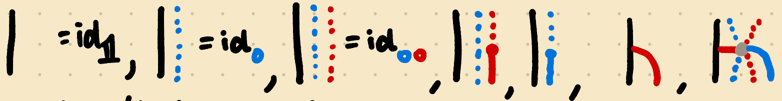
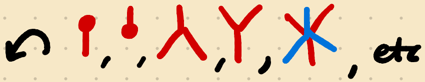
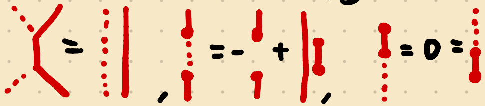
- $\mathcal{D}\tilde{\mathcal{N}} \curvearrowleft \mathcal{D}$:

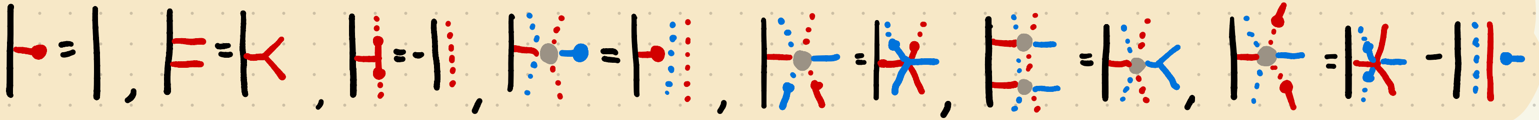
<u>Mor</u> gen by



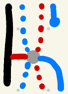


under limited isotopy & local relations

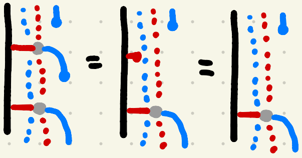


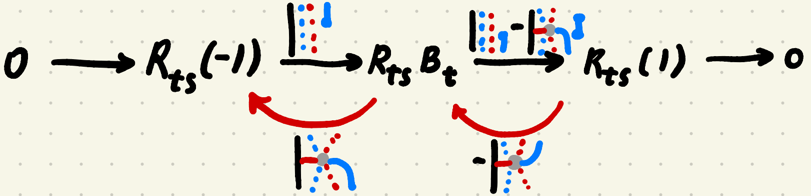


eg.



in an idempotent





2. LV-category diagrammatics

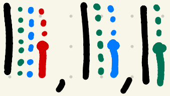
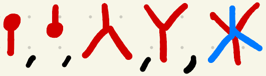
Diagrammatic $\tilde{N}$ for $SU(n,1)$

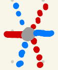
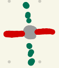
parabolic $S_K = \{s_1, \dots, s_{n-1}\}$

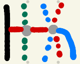
- start w/ $W = S_{n+1}$, $S = \{s_1, \dots, s_n\}$, $W_K = \langle s_1, \dots, s_{n-1} \rangle \leq W$, ${}_{W_K}\backslash W = \{\emptyset, s_n, s_n s_{n-1}, \dots, s_n s_{n-1} \cdots s_1\}$
- Define $\mathcal{D}\tilde{N}$ as right module cat over $\mathcal{D}$

  <u>ob</u> gen by $\mathbb{1}$, open circles for each $s_n s_{n-1} \cdots s_i \in W_K \backslash W$ $\curvearrowleft$ $\mathbb{1}$, $w \in W$

  <u>mor</u> gen by

  id for each object,  etc. for each word in $W_K \backslash W$ $\curvearrowleft$ , etc

   for each $s \in S_K$,  for $m_{st} = 3$,  for $m_{st} = 2$ etc. } composing these horizontally with correct dotted order

  eg 

  under limited isotopy & local relations ...

2. LV-category diagrammatics

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parabolic $S_K = \{s_1, \ldots, s_{n-1}\}$

- Start w/ $W = S_{n+1}$, $S = \{s_1, \ldots, s_n\}$, $W_K = \langle s_1, \ldots, s_{n-1} \rangle \leq W$, $W_K \backslash W = \{\emptyset, s_n, s_n s_{n-1}, \ldots, s_n s_{n-1} \cdots s_1\}$
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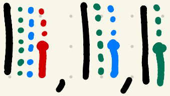
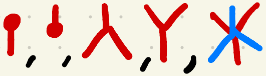
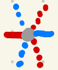
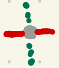
  id for each object,  etc. for each word in $W_K \backslash W$ ↶  , etc

   for each $s \in S_K$,  for $m_{st} = 3$,  for $m_{st} = 2$ etc.

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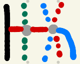
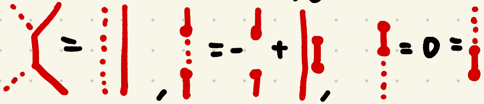
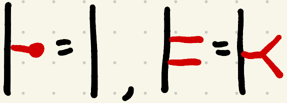
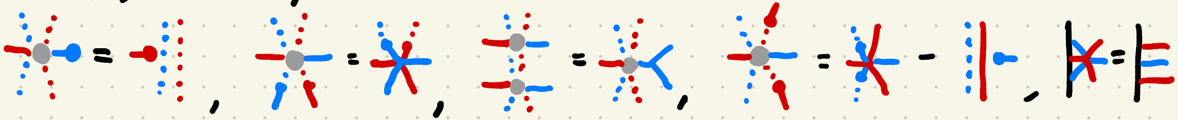
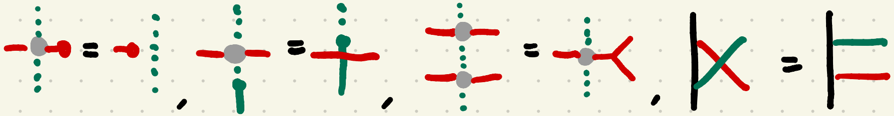
  eg 

  under limited isotopy & local relations

   for each colour in S

   for each colour in S_K

   for $m_{st} = 3$

   for $m_{st} = 2$

2. LV-category diagrammatics

Recap Light leaves & double leaves basis for $\mathcal{D}$

eg. set $W = S_3$, $S = \{s, t\}$, we give an R-basis for $\mathrm{Hom}_{\mathcal{D}}(stss, s)$

light leaves for stss

① Think

step	word	eW	length
0	∅	e	0
1	s	s	1
2	st	st	2
3	sts	sts	3
4	stss	st	2

② For each binary subexpression, assign labels U or D for each component

eg. $\underline{e} = (0, 1, 1, 1)$
U U U D

U = length increases

D = length decreases

0 = don't include

1 = include

③ Inductively define light leaf for $\underline{e}$

if e_i : U1

if e_i : U0

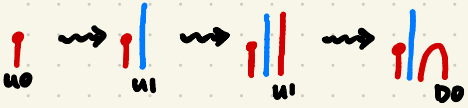
if e_i : D1

if e_i : D0

up to rex moves which we ignore for this talk

eg. $\underline{e} = \overset{U}{(0}, \overset{U}{1}, \overset{U}{1}, \overset{D}{1})$
s t s s

$\mathrm{id}_1 \rightsquigarrow$



$=: LL_{y,\underline{e}}$

2. LV-category diagrammatics

Recap Light leaves & double leaves basis for $\mathcal{D}$

eg. set $W=S_3$, $S=\{s,t\}$, we give an R-basis for $\mathrm{Hom}_{\mathcal{D}}(stss, s)$

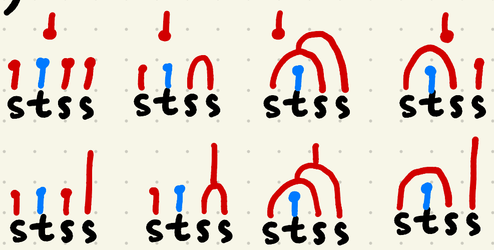
① light leaves for stss

② Light leaves for s

③ Double light leaves basis

- stack light leaves & vertically refl. light leaves of domain & codomain resp.

eg $\mathrm{Hom}(stss, s)$



THM (Libedinsky, Elias-Williamson)

Double light leaves form an R-basis for the appropriate Hom sets.

2. LV-category diagrammatics

Light Leaves in $\mathcal{D}\tilde{N}$

eg. for $SU(2,1)$, $W = S_3$, $S = \{s,t\}$, $W_K = \langle s \rangle \leq W$

we write light leaves for $(t|stt)$

① Take binary subexpr. of st (right side)
and label w/ U, D, X, Y

eg. $t|stt$
011
UDU

eg. $t|stt$
110
UYX

eg. $t|stt$
101
UYX

Taking $W_K \backslash W$ length of whole thing (length of min coset rep)

U : length up

D : length down

X : length same & $\text{last}(\underline{x}) \neq y_i$

Y : length same & $\text{last}(\underline{x}) = y_i$

2. LV-category diagrammatics

Light Leaves in $\mathcal{D}\tilde{W}$

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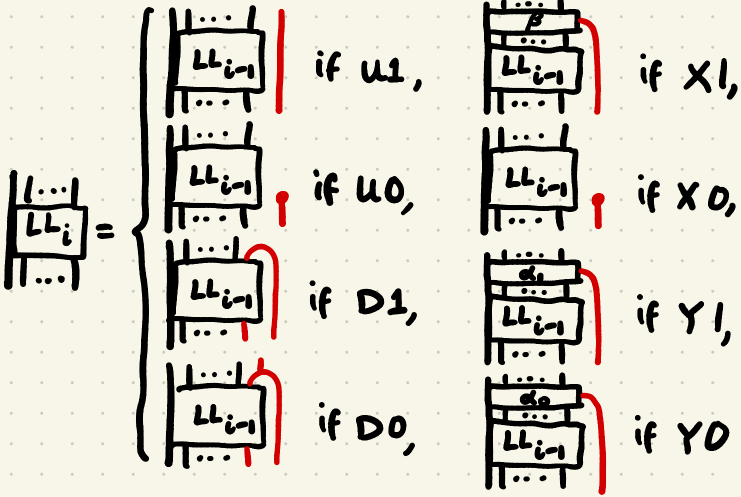
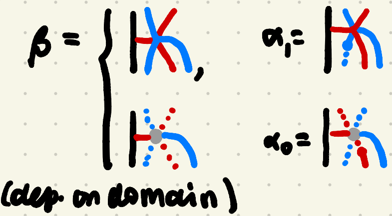
② Induction



for SU(2,1) we have



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eg. for SU(2,1), $W=S_3$, $S=\{s,t\}$, $W_K=\langle s\rangle \leq W$

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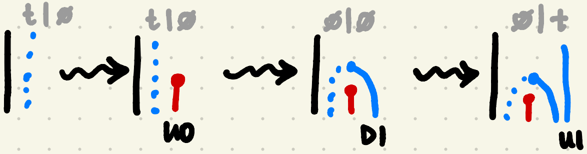
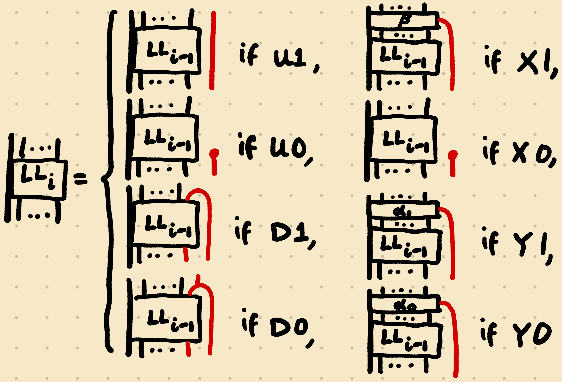
eg. t|stt
0 1 1
U D U



eg. t|stt
1 1 0
U Y X

eg. t|stt
1 0 1
U Y X



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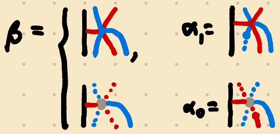

(dep. on domain)

2. LV-category diagrammatics

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eg. for $SU(2,1)$, $W=S_3$, $S=\{s,t\}$, $W_K=\langle s\rangle \leq W$

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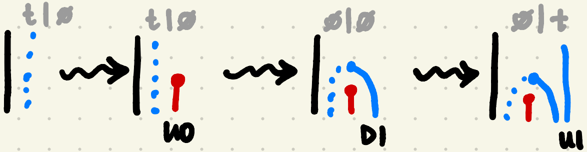
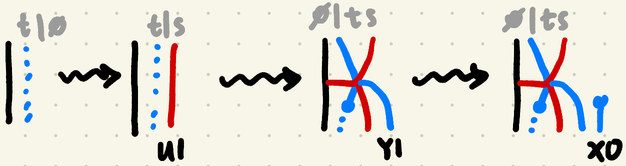
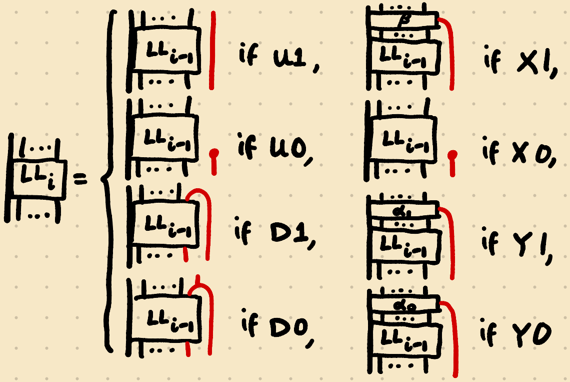


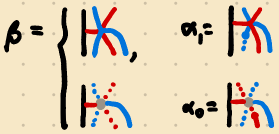
eg. $t|stt$
110
UYX



eg. $t|stt$
101
UYX



for $SU(2,1)$ we have



(dep. on domain)

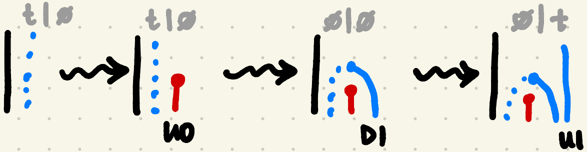
2. LV-category diagrammatics

Light Leaves in $\mathcal{D}\tilde{N}$

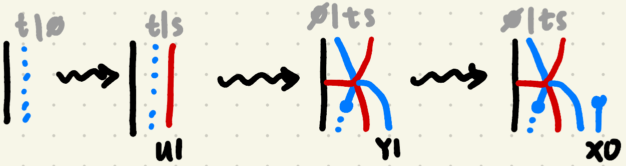
eg. for $SU(2,1)$, $W = S_3$, $S = \{s,t\}$, $W_K = \langle s \rangle \leq W$

we write light leaves for $(t|stt)$

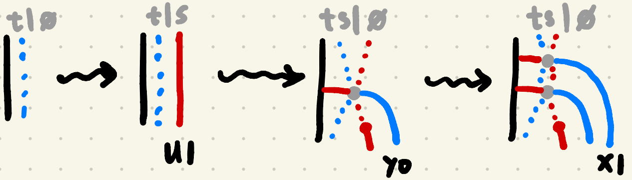
eg. $t|stt$
0 1 1
U D U

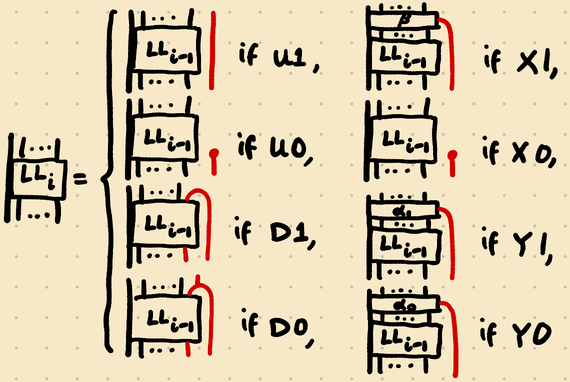


eg. $t|stt$
1 1 0
U Y X

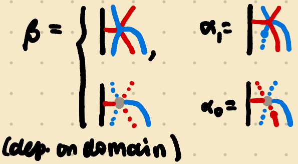


eg. $t|stt$
1 0 1
U Y X





for $SU(2,1)$ we have



2. LV-category diagrammatics

Light leaves in $D\tilde{N}$

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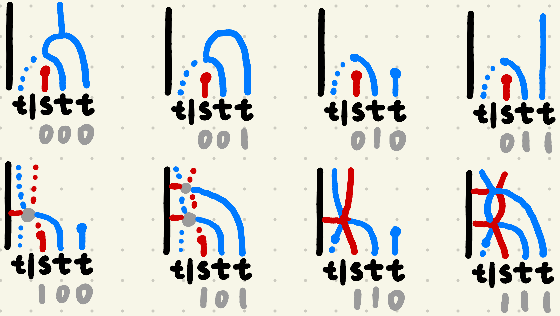
we write light leaves for (t|stt)



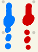
Remark

- In these cases the spherical category (from Tasman's master's thesis) appear as the full subcategory generated by $\mathbb{1}$ ie. if we forget dotted strands
- In this subcategory, Y0, Y1 don't appear and his light leaves aligns with these

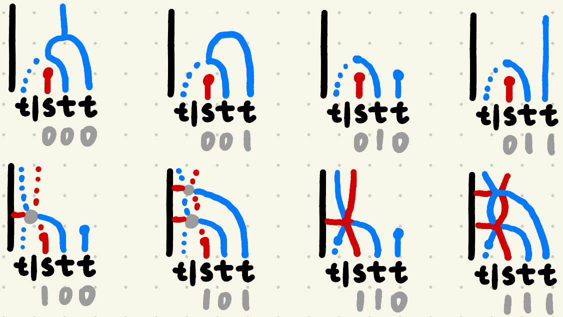
2. LV-category diagrammatics

Double Leaves in $D\tilde{N}$

- similar: compose LL in domain & reflected LL in codomain if they factor through same sequence of colours.

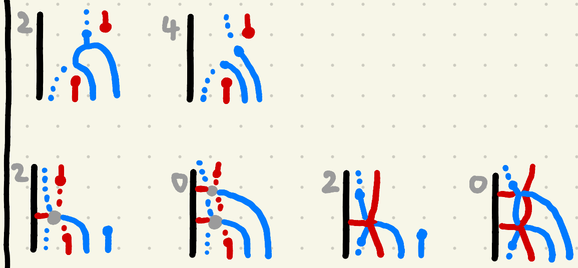
  ie. we can connect  and  by  etc

eg. LL(t|stt)



$\overline{LL}(t|s)$

$\mathbb{LL}(t|stt\ ,\ t|s)$



$\mathrm{rk}^{\bullet}\,\mathrm{Hom}^{\bullet}(t|stt\ ,\ t|s) = v^4 + 3v^2 + 2$

PROP Double light leaves are right R-linearly indep. in the Hom sets in $D\tilde{N}$. (We need to do a slight tweak to YO to prove this)

CONJECTURE Double light leaves in $D\tilde{N}$ form a right R-basis for the respective Hom sets.

that's all