Diagrammatics for a few Lusztig–Vogan Categories

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Motivation

Complex (simply-connected) Lie groups

BGG Category $\mathcal{O}$ $\rightsquigarrow$ Hecke algebra $\mathcal{H}$ $\xrightarrow{\text{categorification}}$ Soergel bimodules $\mathbb{S}\mathrm{Bim}$ $\rightsquigarrow$ Diagrammatics $\mathcal{D}\,\mathbb{S}\mathrm{Bim}$

Real (reductive) Lie groups

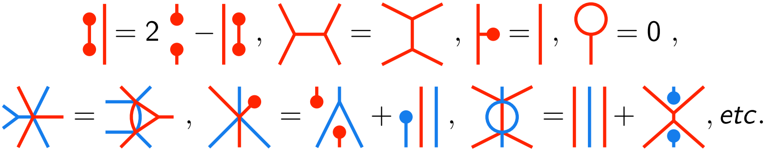
Category of admissible reps. $\rightsquigarrow$ LV-module $M \curvearrowleft \mathcal{H}$ $\xrightarrow{\text{categorification}}$ LV-category $\mathcal{N}^0 \curvearrowleft \mathbb{S}\mathrm{Bim}$ $\overset{?}{\rightsquigarrow}$ Diagrammatics $\mathcal{D}\mathcal{N}^0 \curvearrowleft \mathcal{D}\,\mathbb{S}\mathrm{Bim}$

A reminder of $\mathcal{D}\mathbb{BS}\mathrm{Bim}(\mathfrak{sl}_3)$

$\mathcal{D}\mathbb{BS}\mathrm{Bim}(\mathfrak{sl}_3)$ is a $\mathbb{k}$-linear monoidal category

- Generating objects • and •
- Generating morphisms

- Local relations up to isotopy



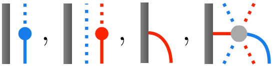
Theorem (Elias–Khovanov 2009)

$\mathcal{D}\mathbb{BS}\mathrm{Bim}(\mathfrak{sl}_3) \simeq \mathbb{BS}\mathrm{Bim}(\mathfrak{sl}_3)$ *as $\mathbb{k}$-linear monoidal categories.*

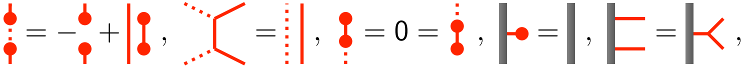
A Glimpse of Diagrammatics for one LV-category

$\mathcal{D}\tilde{\mathcal{N}}^0(SU(2,1))$ is a (right) module category over $\mathcal{D}\mathbb{BS}\mathrm{Bim}(\mathfrak{sl}_3)$

- Generating objects ○ and ○○ $\curvearrowleft \mathbb{1}$, ●, ●, ●●, …
- Generating morphisms (with a left wall)



- Local relations up to (limited) isotopy



Theorem (Z.)

$\mathcal{D}\tilde{\mathcal{N}}^0(SU(2,1)) \simeq \tilde{\mathcal{N}}^0(SU(2,1))$ *as* $\mathbb{k}$*-linear module categories.*